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NUMERICAL STUDY OF THE NONEQUILIBRIUM FILTRATION OF IMMISCIBLE LIQUIDS

O. B. Bocharov, V. V. Kuznetsov, and Yu. V. Chekhovich

UDC 532.546

The uniform displacement of immiscible liquids in a porous medium is studied numerically for two difference methods of accounting for nonequilibrium.

The classical theory of equilibrium filtration of immiscible liquids constructed by Masket and Leverett is based on the function of relative phase permeabilities $K_i(s)$ and the Leverett function J(s), characterizing the capillary pressure jump. These functions were established from experiments on steady-state displacement. The nonsteady displacement process is usually characterized by the presence of regions of sharp change in saturation with respect to both space and time, the Masket—Leverett equilibrium theory sometimes being invalid in this case. The simplest scheme of allowance for nonequilibrium [1] reduces to assuming that the functions $K_1(s)$ and $K_2(s)$, J(s) are the same in a nonequilibrium flow as in the equilibrium case but depend not on the true water saturation s but on a certain effective water saturation σ. The authors of [2] proposed a kinetic equation linking the effective saturation with the true saturation:

$$\sigma = s + \tau(s)\frac{\partial s}{\partial t}. \quad (1)$$

The methods of asymptotic analysis were used in [2] to obtain the dependence of the width of the displacement front on velocity for this model with τ(s) = const, while the problem of countercurrent capillary impregnation was analyzed in [3]. The problem of displacement with large values of the nonequilibrium parameter has not yet been investigated. At the same time, the effect of nonequilibrium on displacement characteristics may be great in the flooding of oil-bearing strata [4].

Here, we use a unidimensional formulation to numerically analyze displacement with a constant total flow of fluids into the sample (Rappaport—Lees problem) within the framework of a model of nonequilibrium filtration with τ(s) monotonically increasing from 0 to τ_* and $s \in [0, s_*]$.

At a constant rate of filtration of the mixture V_0, the system of Masket—Leverett equations and Eq. (1) reduce to the following dimensionless equations (with the same notation being kept for the dimensionless variables):

$$\frac{\partial s}{\partial t} = \frac{\partial}{\partial x}\left[\varepsilon a(\sigma)\frac{\partial \sigma}{\partial x} - F(\sigma)\right], \quad (2)$$

$$DR(s)\frac{\partial s}{\partial t} + s = \sigma. \quad (3)$$

Here, $D = \tau_* V_0/Lm$; $\mu = \mu_1/\mu_2$; $\varepsilon = \gamma\sqrt{Km}/V_0 L\mu_2$; $F(\sigma) = K_1(\sigma)/(K_1(\sigma) + \mu K_2(\sigma))$; $a(\sigma) = -F(\sigma)K_2(\sigma)\frac{dJ(\sigma)}{d\sigma}$;

All-Union Oil and Gas Scientific Research Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 57, No. 1, pp. 91-95, July, 1989. Original article submitted January 4, 1988.

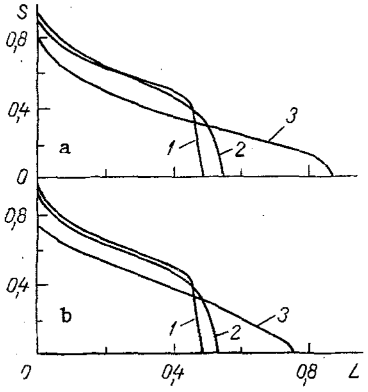

Fig. 1

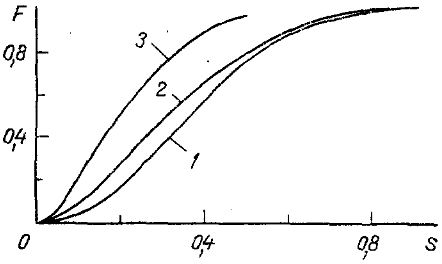

Fig. 2

Fig. 1. Structure of the displacement front with t = 0.3, $\mu = 0$, and $\varepsilon = 0$ (a – with the use of Eq. (3); b – (8)): 1) D = 0 (Baclay–Leverett solution); 2) 0.01; 3) 0.1.

Fig. 2. The relation $F(\sigma(s))$ obtained with the solution of system (2-3) for different values of the nonequilibrium parameter at t = 0.1, $\mu = 1/3$, $\varepsilon = 0$: 1) D = 0; 2) 0.01; 3) 0.1.

$$R(s)=\frac{\tau(s)}{\tau_*}=\begin{cases}\sin\left(\frac{\pi s}{2s_*}\right), & 0\leqslant s<s_*,\\ 1, & s_*\leqslant s\leqslant 1;\end{cases}$$

m is porosity; K is absolute permeability; γ is surface tension; μ_i are the viscosities of the liquids; L is the length of the sample. The time was made dimensionless with respect to the quantity V_0/Lm and the space coordinate with respect to the length L; s is the normalized saturation.

We assign the initial distribution of saturation s(x, 0) = 0 for Eqs. (2) and (3). At the inlet, we assign the condition of equality of the rate of flow of the displacing liquid to the rate of flow of the mixture

$$\varepsilon a(\sigma)\frac{\partial\sigma}{\partial x}-F(\sigma)=-1 \quad \text{at} \quad x=0 \tag{4}$$

We also impose the condition that the flow rate of the liquid at the outlet be proportional to its mobility $K_i(\sigma)/\mu_i$, i.e. we solved the problem without allowance for the end effect

$$\varepsilon a(\sigma)\frac{\partial\sigma}{\partial x}=0 \quad \text{at} \quad x=1. \tag{5}$$

Differentiating (3) with respect to t and inserting $\partial s/\partial t$ from the resulting equation into (2), we arrive at an equation to calculate $\sigma(x, t)$:

$$\frac{\partial\sigma}{\partial t}=\frac{\partial}{\partial x}\left(\varepsilon a(\sigma)\frac{\partial\sigma}{\partial x}-F(\sigma)\right)+D\frac{\partial}{\partial t}\left(R(s)\frac{\partial s}{\partial t}\right). \tag{6}$$

By virtue of the property R(0) = 0, we took zero as the initial value for σ. For Eq. (6), we used the integrointerpolational method in [4] to construct an implicit conservative difference scheme. The term with R(s) $\partial s/\partial t$ on the n-th layer was approximated through σ^n and s^n by virtue of Eq. (3). This allowed us to retain two layers in the scheme. The method of quasilinearization was used to reduce the nonlinear equation for σ^{n+1} to a linear equation. The latter was then solved by the method of three-point trial run. After determining σ_k^{n+1}, we solved Eq. (3) for s by an implicit scheme with iterations for nonlinearity. We then used the resulting values of σ_k^{n+1} and s_k^{n+1} to refine σ^{n+1} and so on until the required accuracy was attained.

The calculations were performed for the model relative phase permeability function and model Leverett function

$$K_1(s) = s^2,\ K_2(s) = (1-s)^2,\ J(s) = \frac{1-s}{3.09(0.1+s)}$$

with $\mu = 1/3$, which corresponds to frontal saturation $s_c = 0.5$ in the Baclay–Leverett theory. Nonequilibrium most characteristically appears at $\varepsilon = 0$. In light of this, we henceforth analyze only this case.

Figure 1a shows profiles of saturation distribution s(x, t) along the sample with different values of the nonequilibrium parameter D. The profiles are compared with the solution of the Baclay–Leverett problem obtained by the same scheme. It was shown in [1, 2] by the methods of asymptotic analysis that even at $\varepsilon = 0$ nonequilibrium leads to blurring of the discontinuity and the formation of a stable transitional zone whose length depends on D.

At large D, the calculations also show a reduction in frontal saturation (Fig. 1a). This is caused by the fact that the lengthwise distribution of saturation is determined by the form of the Baclay–Leverett function – which at large D changes appreciably not only in the region of large saturation gradients near the displacement front, but also in the region as a whole. Figure 2 shows the character of the change in the function F(s) with an increase in the nonequilibrium parameter for the saturation profiles (Fig. 1a). An increase in D accelerates the increase in the function $F(\sigma(s))$ in the region of small degrees of saturation, which in turn leads to a reduction in frontal saturation.

The model of nonequilibrium displacement in [1, 2] is based on the dependence of effective saturation σ on the rate of change in the true saturation. At the same time, effective saturation is also determined by the character of flow of liquid to an elementary volume (particularly in nonuniform media). The inflow depends on the displacement rate and the rate of capillary outflow of liquids into low-permeability blocks, which is determined by the saturation gradient. The simplest relation for determining effective saturation in this case can be written in the form

$$\sigma = s - \tau(s)\left(\frac{\mathbf{v}}{m}\right)\cdot \operatorname{grad} s. \tag{7}$$

Thus, Eq. (1) characterizes the nonequilibrium of the process of displacement through the temporal nonuniformity of saturation at the given point, while Eq. (7) does the same through the spatial nonuniformity of the saturation field. In the unidimensional case, with a constant total filtration velocity at the inlet, Eq. (7) takes the following form in dimensionless variables

$$\sigma = s - DR(s)\frac{\partial s}{\partial x}. \tag{8}$$

A steady-state solution of the travelling wave type, with constant saturation at the displacement front, exists in this case in the same form as for the model [1, 2]. This can be demonstrated by changing over to a coordinate system which moves with the velocity of the displacement front. Then Eq. (8) coincides with (3) to within the coefficients. We used (8) to perform calculations of a displacement problem without allowance for the capillary pressure jump, $\varepsilon = 0$, and with a constant total phase flow at the inlet. Here, R(s) had the same form as previously. To calculate s(x, t) in this case, we obtain the equation

$$\frac{\partial s}{\partial t} + \frac{\partial}{\partial x} F\left(s - DR(s)\frac{\partial s}{\partial x}\right) = 0.$$

The initial condition and the condition at the inlet remained the same. At the outlet we imposed the condition $\partial s/\partial x = 0$, which is equivalent to choosing the phases to be proportional to their mobilities with respect to physical saturation or assuming the absence of nonequilibrium effects at the outlet of the sample. The calculations were performed by a through method on the basis of an explicit scheme which at D = 0 changes to the "explicit angle" scheme for the Baclay–Leverett equation.

Figure 1b shows characteristic profiles of water saturation along the length x obtained in calculations performed for different values of the nonequilibrium parameter D. As in the relaxation model [1, 2] (Fig. 1a), with small values of D (line 2 in Fig. 1b) nonequilibrium redistribution of saturation leads to blurring of the Baclay–Leverett saturation jump

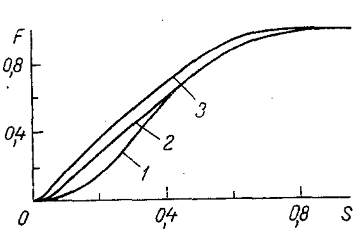

Fig. 3

Fig. 4

Fig. 3. The function $F(\sigma(s))$ obtained in the solution of system (2-8) for different values of the nonequilibrium parameter D with $t = 0.4$, $\mu = 1/3$, $\varepsilon = 0$: 1) $D = 0$; 2) 0.01; 3) 0.1.

Fig. 4. Dependence of the breakthrough time of the displacing phase on the nonequilibrium parameter D calculated from model (2-8); T_0 is the breakthrough time in the Baclay—Leverett problem (with $D = 0$).

(line 1). The length of the stabilized zone increases with an increase in D. At large D (line 3 in Fig. 1a and b), the solutions are also close as a whole but the saturation profiles are flatter in the calculations performed with (8). This is particularly true for large values of saturation.

As is known, to determine frontal saturation in displacement problems it is sufficient to draw a tangent to the curve $F(\sigma(s))$ from the coordinate origin. Then the point of tangency will determine the value of frontal saturation. With an increase in the values of the nonequilibrium parameter, the Baclay—Leverett function will approach linearity within a broad range of saturation s (Fig. 3). Discerning the saturation jump becomes difficult, and the notion of frontal saturation loses meaning. This leads to a flat saturation profile in the solution (line 3 in Fig. 1b) and, thus, to a shortening of the time over which the displacing liquid forces its way out of the sample. It can be seen from Fig. 4 that an increase in D is accompanied by a monotonic decrease in this period of time.

Calculations performed in accordance with the relaxation model [1, 2] showed that the structure of the function $F(\sigma(s))$ does not change qualitatively in the investigated range of the nonequilibrium parameter and frontal saturation can be determined by the established method (see Fig. 2). However, its value decreases appreciably with an increase in D, which leads to shortening of the period of waterless displacement in this model as well. It was shown in [2] that τ_* is on the order of one year for the conditions in oil-bearing strata ($m \sim 0.1$; $L \sim 100$ m; $V_0 \sim 10^{-4}$ cm/sec), which corresponds to $D \sim 3$. The above calculations show that nonequilibrium begins to be significantly manifest at $D = 0.05$. Thus, the nonequilibrium effect may be substantial in strata.

NOTATION

s, true normalized water saturation; σ, effective water saturation; $K_i(s)$, function of relative phase permeabilities; J(s), Leverett capillary pressure function; τ(s), relaxation time; ε, capillary parameter; D, relaxation parameter; μ, ratio of viscosities; F(s), Baclay—Leverett function; V_0, total filtration velocity; t, time; x, space coordinate.

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